Three Dimensional Analysis of Truss Girders by the Thin-Walled Elastic Beam Theory Considering Cross-Sectional Deformations

Yūichirō HAYASHI*, Kōzō HIGUCHI** and Yoshihiro TANAKA***

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**M. Eng., Civil Engineer, Honshu-Shikoku Bridge Authority*
***Civil Engineer, Honshu-Shikoku Bridge Authority*
****M. Eng., Civil Engineer, Honshu-Shikoku Bridge Authority*

Introduction

Honshu-Shikoku Bridges connecting the main island and Shikoku in Japan will have many, simple or 3 span continuous parallel chord truss bridges with about 80~120 m span, each of which carries roadways on a top deck and 4 track railways on a bottom deck. The truss bridge is horizontally folded on its supports, the pier hight is about 50 m and the influence of eccentrically applied live loads is large, therefore, it is feared that the static or dynamic property particularly due to bridge torsion or cross-sectional distortion will appear.

In this paper a method of spatially analyzing a truss bridge has been studied for the beginning of its design, when its bridge width or sway bracing frame is determined after investigating its torsional behavior.

The analytical method used in this paper is fundamentally based on the Vlasov's thin-walled elastic beam theory, having the following features.

(1) A cross section of a truss bridge will be bi-axially symmetric, (2) The displacement method (stiffness method) is used, (3) Since truss member forces and displacements can be always broken down into both a generalized force system (Fig. 2) and a degree of freedom force system (a provisional name) as shown in Fig. 4, the reason of the truss behavior is able to be found, (4) In the analysis, sway bracings will be exist discretely, or as in actual bridges, (5) Coordinate transformation equations for static quantities of the generalized force system have been derived by utilizing those of the degree of freedom force system.

Stiffness Equation for a Straight Truss Bridge

In the first place, a truss bridge is replaced by a box girder in which diagonals and lateral bracings are transformed into thin plates equivalent to shear deformation, and these plate thicknesses are denoted t_h and t_b, respectively. The chord members are regarded as stiffeners at the four corners of the box girder, intermediate or end sway brackings as springs resisting cross-sectional distortion. Then the truss bridge is transformed into such a model as shown in Fig. 1.

If generalized forces and displacements of truss is defined as shown in Fig. 2, eight equations of equilibrium for the part of the box girder intercepted by adjacent sway bracings

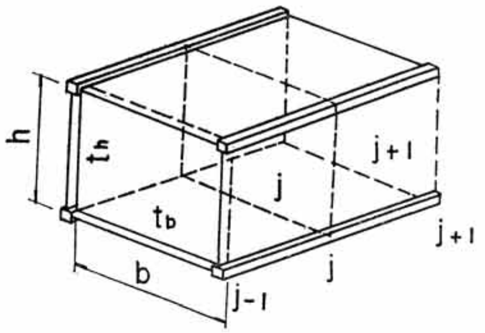

Fig. 1 Thin-walled elastic beam model.

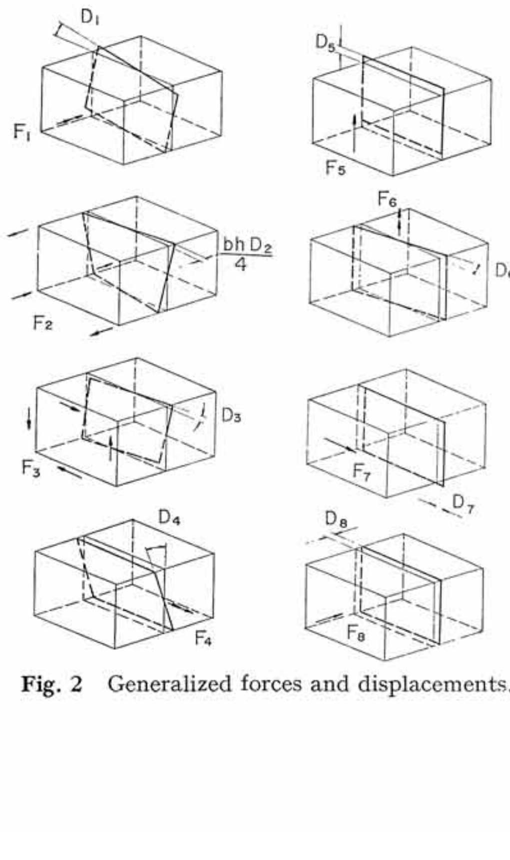

Fig. 2 Generalized forces and displacements.

are obtained due to the Vlasov's theory. Five of them are equations regarding axial force, bending and shear which are generally well known, and the other three regarding torsion, distortion and warping. Three equations regarding the torsions are as follows.

$$a_w D_2'' - b_1 D_2 + b_2 D_1' - b_1 D_3' = 0 \quad \cdots\cdots (1)$$

$$b_2 D_2' - b_1 D_1'' + b_2 D_3'' = 0 \quad \cdots\cdots (2)$$

$$b_1 D_2' - b_1 D_1'' + b_1 D_3'' = 0 \quad \cdots\cdots (3)$$

where

$$a_w = E_s b^2 h^2 A_c / 4, \quad b_1 = G_s (h^2 F_b + b^2 F_h)/2$$

$$b_2 = G_s(-h^2 F_b + b^2 F_h)/2, \quad F_h = h t_h, \quad F_b = b t_b$$

A_c: cross-sectional area of a chord member

The stiffness equation regarding the torsions is obtained from Eqs. (1)~(3) and with boundary conditions. The stiffness equation of the generalized forces $\{F_i^m\}$ $(i=1\sim8)$ and the generalized displacements $\{D_i^m\}$ $(i=1\sim8)$ consist of the above-mentioned stiffness equation, a well-known equation of beam with shear web and that of axial member, expressed as follows.

$$\{F_t^m\} = [K_t^m]\{D_t^m\} \quad \cdots\cdots (4)$$

where

$$\{F_t^m\} = [F_{1a}^m, F_{2a}^m, \cdots, F_{8a}^m, F_{1b}^m, \cdots, F_{8b}^m]^T$$

$$\{D_t^m\} = [D_{1a}^m, D_{2a}^m, \cdots, D_{8a}^m, D_{1a}^m, \cdots, D_{8b}^m]^T$$

a, b: Subscripts distinguishing member ends

The sway bracing rigidity per one frame Γ_B is obtained from the following equation for the deformation as shown in Fig. 3.

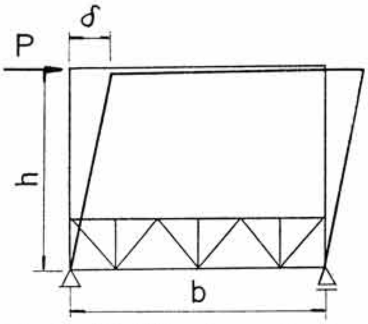

Fig. 3 Rigidity of sway bracing.

$$\Gamma_B = F_3/D_3 = 4h^2 P/\delta \quad \cdots\cdots (5)$$

which becomes D_3-element of the sway bracing stiffness matrix $[R^m]$ (8×8). If the degree of freedom forces $\{f^m\}$ and degree of freedom displacements $\{d^m\}$ are defined as shown in Fig. 4 in order to easily process the boundary conditions on the supports, the coordinate transformation equations between that system and the generalized force system are given as follows.

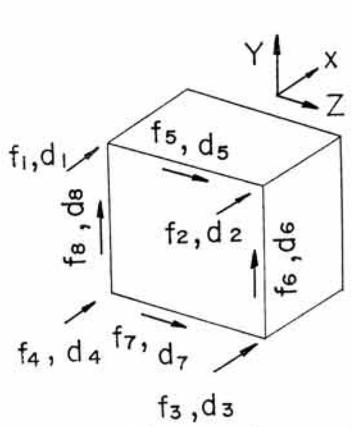

Fig. 4 Degree of freedom forces and displacements.

$$\{f^m\} = [T_1]\{F^m\} \quad \cdots\cdots (6)$$

$$\{D^m\} = [T_1]^T\{d^m\} \quad \cdots\cdots (7)$$

where

$$[T_1] = \begin{bmatrix} 0 & -1/bh & 0 & -1/2h & 0 & -1/2b & 0 & 1/4 \\ 0 & 1/bh & 0 & -1/2h & 0 & 1/2b & 0 & 1/4 \\ 0 & -1/bh & 0 & 1/2h & 0 & 1/2b & 0 & 1/4 \\ 0 & 1/bh & 0 & 1/2h & 0 & -1/2b & 0 & 1/4 \\ 1/2h & 0 & 1/2h & 0 & 0 & 0 & 1/2 & 0 \\ -1/2b & 0 & 1/2b & 0 & 1/2 & 0 & 0 & 0 \\ -1/2h & 0 & -1/2h & 0 & 0 & 0 & 1/2 & 0 \\ 1/2b & 0 & -1/2b & 0 & 1/2 & 0 & 0 & 0 \end{bmatrix}$$

then the equlibrium equation expressed with the degree of freedom force system at j-cross section becomes

$$\{f_{tj}^m\}_b + \{f_{tj+1}^m\}_a + [\gamma_j^m]\cdot\{d_j^m\} = \{\Delta P_j\} \quad \cdots\cdots (8)$$

where

$$\{f_t^m\} = [f_{1a}^m, f_{2a}^m, \cdots, f_{8a}^m, f_{1b}^m, \cdots, f_{8b}^m]^T$$

$$[\gamma^m] = [T_1][R^m][T_1]^T$$

$\{\Delta P\}$: External force vector

Using the boundary conditions for Eq. (8), the solutions of the displacements and forces in the degree of freedom force system are obtained with the displacement method. Transformation of the degree of freedom forces into truss member forces are performed considering the property proper to truss bridges, but the details have been omitted here.

Stiffness Equation for Folded Truss Bridge

Coordinate transformation equations for a folded truss bridge as shown in Fig. 5 are derived in the order as shown in Fig. 6, and the coordinate transformation matrixes $[T_F]$ and $[T_D]$ in the generalized force system are

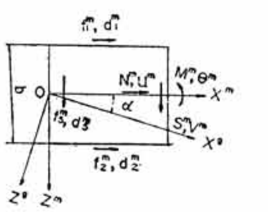

Fig. 5 Folded bridge.

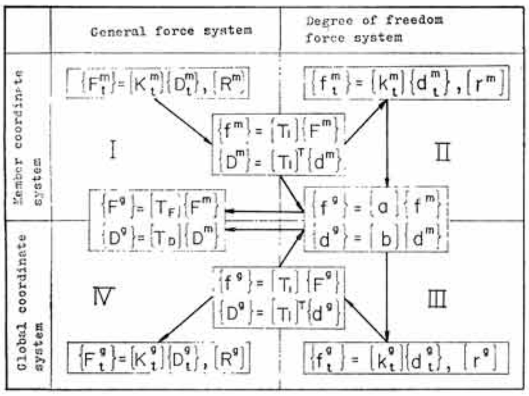

Fig. 6 Derivation of coordinate transformation matrix.

$[T_F]$

$$=\begin{bmatrix} (\cos\alpha+1)/2 & 0 & (\cos\alpha-1)/2 & \sin\alpha & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ (\cos\alpha-1)/2 & 0 & (\cos\alpha+1)/2 & \sin\alpha & 0 & 0 & 0 & 0 \\ -(\sin\alpha)/2 & 0 & -(\sin\alpha)/2 & \cos\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cos\alpha & -\sin\alpha \\ 0 & 0 & 0 & 0 & 0 & 0 & \sin\alpha & \cos\alpha \end{bmatrix} \quad \cdots\cdots(9)$$

$$[T_D]=[[T_F]^T]^{-1} \quad \cdots\cdots(10)$$

and the other transformation equations shown in Fig. 6 have been omitted here.

Numerical Analysis

The accuracy of the authors' method (denoted A–M) is examined by comparing the numerical values from A–M with those from the so-called matrix displacement method considering individual members (denoted MD–M) for the folded truss bridge shown in Fig. 7 with the load condition shown in Fig. 8. The axial forces of the top chord members and sway bracing members are shown in Fig. 9 and Fig. 10. The other examples have been omitted here. This results show the good accuracy of A–M.

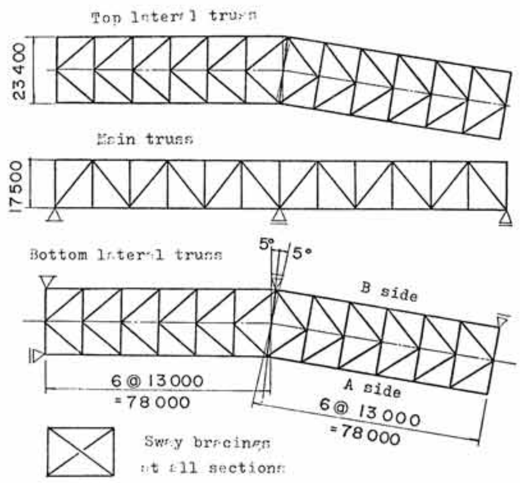

Fig. 7 Folded truss bridge model.

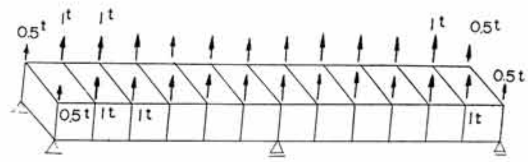

Fig. 8 Load condition.

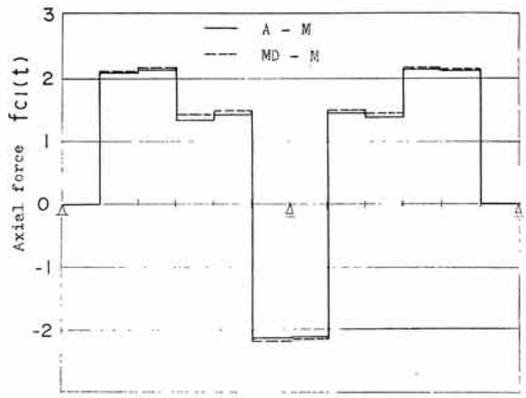

Fig. 9 Axial force of B-side top chord.

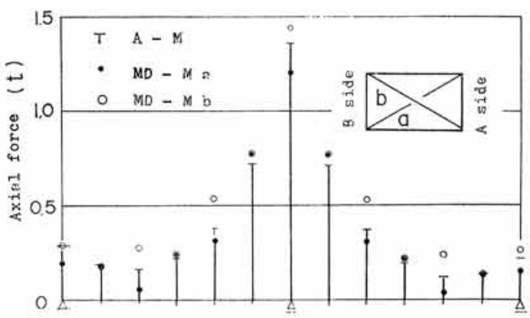

Fig. 10 Axial force of sway bracing member.