Presentation of the Cash Flow Matrix (CF Matrix) Analysis for National Accounts and a Consideration of the Leontief Input Output Analysis Theory

No multiplier effect is found in Leontief Input-Output Analysis

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Abstract

The purpose of the Input Output (IO) analysis by W.W.Leontief is to analyze the production account between industrial sectors in the System of National Accounts (SNA). That of the CF matrix analysis presented by the author is to provide a cash flow matrix analysis method, in the SNA, where the 3 accounts (production, consumption and accumulation accounts) and the financial account are combined to become a matrix-vector equation. The first basic equation in the CF matrix analysis is the same as the basic production equation in the IO analysis. The second basic equation in the CF matrix analysis corresponds to the price model equation in the IO analysis. It is revealed that the latter equation is mistaken. An Affine transformation equation (a mapping relationship between vectors) joining the first and second equations in the CF matrix analysis is derived. This shows that the Leontief inverse matrix solution in the IO analysis shows an expansion of one industry's economic value of final goods to other industries through the transactions with them; and that the final goods value ends up converging at the gross value added.

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1 Introduction

This theory was born when the author applied his original managed gross profit chart theory [1] to Wassily Leontief's input-output analysis theory [2] in order to ascertain the effectiveness of the author's profit chart theory in economies. This produced a break-even chart for national accounts [3]. The break-even chart showed marked differences to the investment multiplier effect chart shown by J. M. Keynes. The author took this opportunity to begin to investigate the truth of Keynes' investment multiplier effect theory. He has since demonstrated, through individual case-studies, that the multiplier effect theory is mathematically mistaken [4]].

According to the managed gross profit chart theory, no economic values due to the multiplier effect can be comprised in national accounts (**SNA**) and neither therefore in the input-output table. For this reason, the basic production equation must include, in a pair of production equations, another hitherto unbeknown equation causing an inverse multiplier operation against the multiplier effect. Upon further analysis the author found the said unbeknown equations and named the analysis the 'CF matrix analysis'. He tentatively published the result on his website [5] in Dec. 2003. At that time it had not been realized that the equation could be ground-breaking enough to replace the input output analysis altogether.

Subsequently, the extension of the managed gross profit chart theory, the logic behind the involuntary unemployment problem by J.M.Keynes, the innovation theory by J. A. Schumpeter and the disproof of the general equilibrium theory by M. E. L. Walras [8] were also researched, and some successful results were presented [6], [7]. However, he has not thus far been able to present a simple and clear unified theory to replace those of Walras and Keynes.

The scale of monetary economies has recently become overwhelmingly larger than that of object economies, such that the former is now disturbing the latter. In 2008, a financial crisis stemming from the bankruptcy of an investment company in the USA rippled through financial industries all over the world, even reaching object economies. As a result, unemployment levels have increased the world over. The author thought that the CF matrix analysis might prove useful in resolving the issue of unemployment, and resumed research into the CF matrix analysis. As a result, he has found mistakes in the input output analysis by Leontief and created a methodology for the CF matrix analysis as a first step.

Though the CF matrix analysis theory has produced such contents and analytical results, several problems do remain; for example, the issue of how to treat loans provided by deposit money banks including the central bank. Therefore the theory is not yet fully complete. However, the author can now clearly highlight the errors included in the input output analysis such as the error of the multiplier effect of Leontief's inverse matrix solution. Regarding the present as an appropriate moment, this paper has now been published.

2 Building the CF matrix analysis method

2.1 Basic equation expressing sales transactions in the SNA

We will now consider industrial transaction accounts in an input output table. The horizontal matrix transformation method in the direction in the CF matrix analysis is the same as that of the output process in the input output analysis originally presented by W.W. Leontief. For the development of the author's theory, Leontief's formulation is shown. Using notations shown in Chapter 5, each industry's transaction account between 3 industrial sectors is shown in the following:

Account 1

Sector 1		Sector 2		Sector 3	
Dr.	Cr.	Dr.	Cr.	Dr.	Cr.
P_{11}	P_{11}	P_{12}	P_{21}	P_{13}	P_{31}
P_{21}	P_{12}	P_{22}	P_{22}	P_{23}	P_{32}
P_{31}	P_{13}	P_{32}	P_{23}	P_{33}	P_{33}
V_1	Y_1	V_2	Y_2	V_3	Y_3
X_1	X_1	X_2	X_2	X_3	X_3

where P_{ij}=material sales from i industry to j industry or material purchases from j industry to i industry; Y_i=final product +export+incremental inventory, of i industry; V_i=GVA+import, of i industry; X_i= total of credit or debit in i sector's account.

The graphic cash flow of Account 1 is shown in Fig.1. The order of the suffix, 'ij' expressing the cash flow is inverse to that of goods flow.

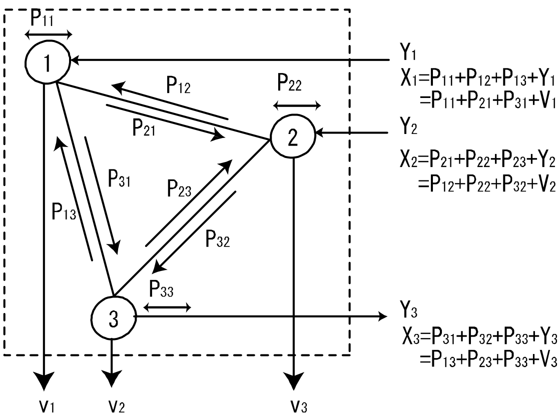

Fig.1 Graphic expression of Account1

Account 1 can be transformed into the input output table Fig.2. Since Account 1 and Fig.2 are equivalent, no assumptions are needed other than the following in the CF matrix analysis: *Account 1 should be made from the industrial aggregate calculation made from the individual enterprise final income statements.*

industry	1	2	3	a	b
1	P_{11}	P_{12}	P_{13}	Y_1	X_1
2	P_{21}	P_{22}	P_{23}	Y_2	X_2
3	P_{31}	P_{32}	P_{33}	Y_3	X_3
a	V_1	V_2	V_3		
b	X_1	X_2	X_3		

Fig.2 CF matrix table of Account 1

where we name $\mathbf{P} = [P_{ij}]\,(i, j = 1, 2, \cdots)$ the **P matrix** or the **intermediate goods matrix**.

Leontief takes his stand on Walras's general equilibrium theory, while the author takes the stance that no general equilibrium is in production. Further, since the author's purpose is to establish cash flow analyses for overall economies, he occasionally coins new technical terms in place of those used in Leontief's theory. In the horizontal (row) direction in Fig.2 the following equations hold:

$$
\begin{aligned}
P_{11} + P_{12} + P_{13} + Y_1 &= X_1 \longrightarrow \text{horizontal direction} \\
P_{21} + P_{22} + P_{23} + Y_2 &= X_2 \longrightarrow \text{horizontal direction} \\
P_{31} + P_{32} + P_{33} + Y_3 &= X_3 \longrightarrow \text{horizontal direction}
\end{aligned} \tag{1}
$$

These equations are transformed:

$$
\begin{aligned}
(P_{11}/X_1)X_1 + (P_{12}/X_2)X_2 + (P_{13}/X_3)X_3 + Y_1 &= X_1 \\
(P_{21}/X_1)X_1 + (P_{22}/X_2)X_2 + (P_{23}/X_3)X_3 + Y_2 &= X_2 \\
(P_{31}/X_1)X_1 + (P_{32}/X_2)X_2 + (P_{33}/X_3)X_3 + Y_3 &= X_3
\end{aligned} \tag{2}
$$

Here a_{ij} is defined as:

$$a_{ij} = P_{ij}/X_j \tag{3}$$

where a_{ij} is the **intermediate sales proportion coefficients**.

Eq.(2) is expressed as follows:

$$
\begin{aligned}
a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + Y_1 &= X_1 \\
a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + Y_2 &= X_2 \\
a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + Y_3 &= X_3
\end{aligned} \tag{4}
$$

The coefficient a_{ij} is a constant giving the proportion of P_{ij} to X_j. Since a_{ij} in each equation in Eq.(4) is made from numerical values (constants) in financial statements, no assumption for a_{ij} is required.

Transforming Eq.(4) into a matrix form gives the following:

$$\mathbf{AX} + \mathbf{Y} = \mathbf{X} \tag{5}$$

where $\mathbf{A} = [a_{ij}]\,(i,j = 1,2,\cdots)$ is named the **A matrix** or the **intermediate sales proportion matrix**. $\mathbf{Y} = [Y_1, Y_2, \cdots]^T$ and $\mathbf{X} = [X_1, X_2, \cdots]^T$ are vectors.

From Eq.(5) we have:

$$\mathbf{Y} = [\mathbf{E} - \mathbf{A}]\,\mathbf{X} \tag{6}$$

$$\mathbf{X} = [\mathbf{E} - \mathbf{A}]^{-1}\,\mathbf{Y} \tag{7}$$

where **E**=unit matrix. Matrix $[\mathbf{E} - \mathbf{A}]^{-1}$ is called the **Leontief inverse matrix**. We name Eq.(6) or Eq.(7) the **sales transaction equation** in the CF matrix analysis. Thus far derivations have followed Leontief's IO analysis.

2.2 Another basic equation expressing purchase transactions

In the vertical (column) direction in Fig.2 the following equations hold:

$$\begin{aligned} P_{11} + P_{21} + P_{31} + V_1 &= X_1 \downarrow \text{vertical direction} \\ P_{12} + P_{22} + P_{32} + V_2 &= X_2 \downarrow \text{vertical direction} \\ P_{13} + P_{23} + P_{33} + V_3 &= X_3 \downarrow \text{vertical direction} \end{aligned} \tag{8}$$

These equations can be transformed to:

$$\begin{aligned} (P_{11}/X_1)X_1 + (P_{21}/X_2)X_2 + (P_{31}/X_3)X_3 + V_1 &= X_1 \\ (P_{12}/X_1)X_1 + (P_{22}/X_2)X_2 + (P_{32}/X_3)X_3 + V_2 &= X_2 \\ (P_{13}/X_1)X_1 + (P_{23}/X_2)X_2 + (P_{33}/X_3)X_3 + V_3 &= X_3 \end{aligned} \tag{9}$$

By defining the notation in Eq.(10) as in Eq.(3), Eq.(9) is transformed to Eq.(11):

$$b_{ij} = P_{ji}/X_j \tag{10}$$

$$\begin{aligned} b_{11}X_1 + b_{12}X_2 + b_{13}X_3 + V_1 &= X_1 \\ b_{21}X_1 + b_{22}X_2 + b_{23}X_3 + V_2 &= X_2 \\ b_{31}X_1 + b_{32}X_2 + b_{33}X_3 + V_3 &= X_3 \end{aligned} \tag{11}$$

We name b_{ij} the **intermediate purchase proportion coefficients**. From both Eq.(2) and Eq.(9) as well as Numerical calculation example 1, it can be easily ascertained that $[b_{ij}] \neq [a_{ij}]^T$.

Defining the vector $\mathbf{V} = [V_1, V_2, \cdots]^T$, and referring to Eq.(5), Eq.(11) is expressed as:

$$\mathbf{BX} + \mathbf{V} = \mathbf{X} \tag{12}$$

where $\mathbf{B} = [b_{ij}]\,(i, j = 1, 2, \cdots)$ is named the **B matrix** or the **intermediate purchase proportion matrix**. Thus we obtain the following:

$$\mathbf{V} = [\mathbf{E} - \mathbf{B}]\,\mathbf{X} \tag{13}$$

$$\mathbf{X} = [\mathbf{E} - \mathbf{B}]^{-1}\,\mathbf{V} \tag{14}$$

Eq.(12) or Eq.(14) is named the **Purchase transaction equation** in the CF matrix analysis.

2.3 Joining the sales and purchase transaction equations

From Eq.(6) and Eq.(14) we have:

$$\mathbf{Y} = [\mathbf{E} - \mathbf{A}]\,[\mathbf{E} - \mathbf{B}]^{-1}\,\mathbf{V} \tag{15}$$

Thereby defining the following notation **H**, and naming it the **H** matrix,:

$$\mathbf{H} = [\mathbf{E} - \mathbf{A}]\,[\mathbf{E} - \mathbf{B}]^{-1} \tag{16}$$

Eq.(15) is expressed as:

$$\mathbf{Y} = \mathbf{HV} \tag{17}$$

Further we have:

$$\mathbf{V} = \mathbf{H}^{-1}\mathbf{Y} \tag{18}$$

The relation between **Y** and **V** is now been established. We name Eq.(12) or Eq.(14) the **joined transaction equation**.

What functional relation, then, holds between **P**, **Y** and **V?** If we consider **Y** and **V** as exogenous variables, we have the following; no equations can be obtained.

$$\begin{aligned}\mathbf{AX} &= \mathbf{X} - \mathbf{Y} \\ &= [\mathbf{E} - \mathbf{A}]^{-1}\,\mathbf{Y} - \mathbf{Y}\end{aligned} \tag{19}$$

$$\begin{aligned}\mathbf{BX} &= \mathbf{X} - \mathbf{V} \\ &= [\mathbf{E} - \mathbf{B}]^{-1}\,\mathbf{V} - \mathbf{V}\end{aligned} \tag{20}$$

These two equations are meaningless. Since Eq.(5) and Eq.(12) simply represent the relation, total value = exogenous variable values+endogenous variable values. They simply represent that **AX** and **BX**=endogenous variable values. We shall regard the CF matrix analysis as a cash flow analysis in economies. Conclusively speaking, while it may seem that there is a functional relation between **P**, **Y** and **V** in Eq.(5), **P** is really indeterminate to both **Y** and **V**. Economic values are not included in **P**, therefore **P** shows only flow patterns of economic values included in **V** and **Y**; an infinite number of combinations of **P** elements can be found. As this is not easily demonstrated through theoretical analysis alone, it will be further explained with Numerical calculation example 3.

2.4 Numerical calculation examples for the CF matrix analysis

2.4.1 Numerical calculation example 1

Since the CF matrix analysis is a new analytical method, numerical calculation examples themselves have their own significance. The CF matrix analysis pulls apart traditional economic common sense established by the input output analysis. In the input output analysis the solvability with non-negative solutions for Eq.(7) becomes subject to debate, known as the Hawkins-Simon condition. In the CF matrix analysis, such a condition does not come into question. The CF matrix table shown in Fig.1 is equivalent to Account 1; the CF matrix table, including any of its transformations, is no more than Account 1 from a different angle. If Account 1 is practical, any analytical results obtained using the CF matrix analysis must therefore be practical.

Fig.3 shows an example in which V item value in sector 1 is taken as −50. Although this example may be an unlikely situation under real codition, it has been intentionally adopted. We can examine $\mathbf{B} \neq \mathbf{A}^T$ in this example.

	1	2	3	Y	X
1	10	20	30	40	100
2	50	60	70	80	260
3	90	100	110	120	420
V	**−50**	80	210		
X	100	260	420		

Fig.3 Numerical calculation example 1

$$\mathbf{A} = \begin{bmatrix} 10/100 & 20/260 & 30/420 \\ 50/100 & 60/260 & 70/420 \\ 90/100 & 100/260 & 110/420 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 10/100 & 50/260 & 90/420 \\ 20/100 & 60/260 & 100/420 \\ 30/100 & 70/260 & 110/420 \end{bmatrix}$$

$$\mathbf{H} = [\mathbf{E} - \mathbf{A}]\,[\mathbf{E} - \mathbf{B}]^{-1} = (1/956) \begin{bmatrix} 1136 & 306 & 336 \\ -357 & 851 & -45 \\ -894 & -516 & 530 \end{bmatrix}$$

$$\mathbf{HV} = (1/956) \begin{bmatrix} 1136 & 306 & 336 \\ -357 & 851 & -45 \\ -894 & -516 & 530 \end{bmatrix} \begin{bmatrix} -50 \\ 80 \\ 210 \end{bmatrix} = \begin{bmatrix} 40 \\ 80 \\ 120 \end{bmatrix} = \mathbf{Y}$$

2.4.2 Numerical calculation example 2

In this example only the diagonal elements in Fig.3 have been replaced by 0. Matrix **H** in Fig.3 became equal to that in Fig.4. We obtain the same result regardless of figures entered in the diagonal elements. This shows that any self-dealing transaction does not have an effect on external transactions. We can make sense of the reason for this in Fig.12.

	1	2	3	Y	X
1	**0**	20	30	40	90
2	50	**0**	70	80	200
3	90	100	**0**	120	310
V	−50	80	210		
X	90	200	310		

Fig.4 Numerical calculation example 2

$$\mathbf{A} = \begin{bmatrix} 0 & 20/200 & 30/310 \\ 50/90 & 0 & 70/310 \\ 90/90 & 100/200 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 50/200 & 90/310 \\ 20/90 & 0 & 100/310 \\ 30/90 & 70/200 & 0 \end{bmatrix}$$

$$\mathbf{H} = [\mathbf{E} - \mathbf{A}]\,[\mathbf{E} - \mathbf{B}]^{-1} = (1/956) \begin{bmatrix} 1136 & 306 & 336 \\ -357 & 851 & -45 \\ -894 & -516 & 530 \end{bmatrix}$$

2.4.3 Numerical calculation example 3

This example shows that the matrix **P** is independent of both vectors **Y** and **V**; we have an infinite number of solutions which satisfy the relation between **Y** and **V**; thus, the solution of **P** is indefinite. Since this example relates to Leontief price model analysis, it will be taken up again later .

Fig. 5 shows the input output table of Japan, in a year, made up of 4 sectors, which has been re-formed from the 13 sector table by the author.

	1	2	3	4	$\Sigma_{j=1}^{4} P_{ij}$	Y_i	X_i
1	**3**	17	5	4	29	**77**	106
2	29	**125**	6	36	196	**119**	315
3	9	29	**25**	30	93	**136**	229
4	14	33	24	**43**	114	**173**	287
$\Sigma_{i=1}^{4} P_{ij}$	55	204	60	113			
V_j	**51**	**111**	**169**	**174**			
X_j	106	315	229	287			

Unit:10 billion dollars, 1$=100¥

Fig.5 Input output table in Japan

Fig.6 illustrates that we can easily make **P** with arbitrary figures of elements fixing **Y** and **V** as constant element vectors. The number of unknowns from P_{11} to P_{44} is 16 in **P**. We can set up 8 equations from the 8 accounts in the horizontal and vertical directions. We know that replacing diagonal element figures with 0 does not change its analytical result. If we enter 0 into the diagonal elements, the number of unknowns becomes 12, and we have Fig.6:

	1	2	3	4	$\Sigma_{j=1}^{4} P_{ij}$	Y	X
1	**0**	P_{12}	P_{13}	P_{14}	26	**77**	103
2	P_{21}	**0**	P_{23}	P_{24}	71	**119**	190
3	P_{31}	P_{32}	**0**	P_{34}	68	**136**	204
4	P_{41}	P_{42}	P_{43}	**0**	71	**173**	244
$\Sigma_{i=1}^{4} P_{ij}$	52	79	35	70			
V	**51**	**111**	**169**	**174**			
X	103	190	204	244			

Fig.6 Indeterminateness of **P**

In Fig.6 if we give 4 unknowns arbitrary constants, the number of unknowns and equations falls to 8, and we will be able to obtain solutions. Taking $P_{14} = 10$, $P_{24} = 20$, $P_{31} = 30$, $P_{34} = 40$ and $P_{43} = 50$ gives Fig.7.

	1	2	3	4	$\Sigma_{j=1}^{4} P_{ij}$	Y	X
1	**0**	P_{12}	P_{13}	**10**	26	**77**	103
2	P_{21}	**0**	P_{23}	**20**	71	**119**	190
3	**30**	-2	**0**	**40**	68	**136**	204
4	P_{41}	P_{42}	**50**	**0**	71	**173**	244
$\Sigma_{i=1}^{4} P_{ij}$	52	79	35	70			
V	**51**	**111**	**169**	**174**			
X	103	190	204	244			

Fig.7 Indeterminateness of **P**

In Fig. 7 we automatically obtain $P_{32} = -2$. Further we have $P_{12} + P_{13} = 26 - 10 = 16$, $P_{21} + P_{23} = 51$, $P_{41} + P_{42} = 21$, $P_{21} + P_{41} = 2\ 2$, $P_{12} + P_{42} = 81$ and $P_{13} + P_{23} = -15$. These relationships are expressed with the following simultaneous equations:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_{12} \\ P_{13} \\ P_{21} \\ P_{23} \\ P_{41} \\ P_{42} \end{bmatrix} = \begin{bmatrix} 16 \\ 51 \\ 21 \\ 22 \\ 81 \\ -15 \end{bmatrix}$$

Taking $P_{42} = 60$ so that $|[P_{ij}]| \neq 0$, we have the solution $[P_{12}, P_{13}, P_{21}, P_{23}, P_{41}, P_{42}]^T = [21, -5, 61, -10, -39, 60]^T$ and therefore Fig.8 results.

	1	2	3	4	$\Sigma_{j=1}^{4} P_{ij}$	Y	X
1	**0**	21	-5	**10**	26	**77**	103
2	61	**0**	-10	**20**	71	**119**	190
3	**30**	-2	**0**	**40**	68	**136**	204
4	-39	**60**	**50**	0	71	**173**	244
$\Sigma_{i=1}^{4} P_{ij}$	52	79	35	70			
V	**51**	**111**	**169**	**174**			
X	103	190	204	244			

Fig.8 Indeterminateness of **P**

3 Consideration of structure and problems in Leontief input output analysis theory

3.1 The open Leontief system

There are two systems in Leontief input output analysis, the closed model system and the open model system. These are described later.

The author intends to express a cash flow analysis to include financial credit transactions in economies, whereas the Leontief input output analysis targets and presents an analysis for production processes of real goods. Therefore the two analytical objectives largely differ, but because the author has referred to Leontief's data creation method from economic accounts, the two methods have many similarities. Thus, the author's data creation method will be explained in Part 2. Here Leontief open system, a standard analytical theory in modern economics, will be reviewed and reconsidered.

The main assumptions in Leontief input output analysis are shown in the following:

Assumption a1 Each industry has but one product and one means of production.

Assunption a2 Each input cost in an industry is proportional to its input quantity from its own and other industries.

Assumption a3 Each output is produced via a unique combination of inputs.

Assumption a4 There exists 'Constant returns to scale' in each production process.

Assumption a5 All input costs are consumed in all output production processes.

Assumption a6 There is no debtor-creditor relationship in production processes.

Leontief's input output analysis will be described based on present standard texts on the theory. The notation of subscripts is the same as in general input output analyses. Note the i row, including (i,i) element in Fig.2. Each $P_{ij}(j=1,2,\cdots)$ represents the sales or outputs from industry i's intermediate goods to industry j; Y_i is the sales of industry i's final goods. Note the i column, including (i,i) element. Each $P_{ji}(j=1,2,\cdots)$ is industry i's purchases or inputs of intermediate goods from industry j. V_i is the industry i's own input of the gross value added to itself. In addition, (i,i) element or an element in the diagonal elements in **P** expresses sales (purchases are allowable) within industry i. Any value for (i,i) element does not effect any analytical results.

Let the quantity (quantitative unit such as ton) of intermediate goods sold from industry i to industry j be denoted q_{ij}, that of final goods be denoted y_i, and that of the total quantity of output be denoted q_i. Suppose that own consumption q_{ii} is taken into consideration. Since the difference between industry i's total output quantity and total input quantities from the other industries equals industry i's final goods quantity, we have the following, when the number of industrial sectors= 3:

$$\begin{aligned} q_{11} + q_{21} + q_{31} + y_1 &= q_1 \\ q_{12} + q_{22} + q_{32} + y_2 &= q_2 \\ q_{13} + q_{23} + q_{33} + y_3 &= q_3 \end{aligned} \tag{L1}$$

For example, each input $q_{i1}(i = 1, 2, 3)$ from industry i to industry 1 in the first equation in Eq.(L1) will be determined by an industry i's technical structure. Letting the input quantity for industry j to purchase from industry i in order to produce the industry j's one unit output be denoted a_{ij}, we have:

$$a_{ij} = q_{ij}/q_j \tag{L2}$$

where a_{ij} is a constant from Assumption a2.

Eq. (L1) can be transformed to:

$$\begin{aligned} (q_{11}/q_1)q_1 + (q_{21}/q_2)q_2 + (q_{31}/q_3)q_3 + y_1 &= q_1 \\ (q_{12}/q_1)q_1 + (q_{22}/q_2)q_2 + (q_{32}/q_3)q_3 + y_2 &= q_2 \\ (q_{13}/q_1)q_1 + (q_{23}/q_2)q_2 + (q_{33}/q_3)q_3 + y_3 &= q_3 \end{aligned} \tag{L3}$$

Substituting Eq.(L2) into Eq.(L3) gives:

$$\begin{aligned} a_{11}q_1 + a_{21}q_2 + a_{31}q_3 + y_1 &= q_1 \\ a_{12}q_1 + a_{22}q_2 + a_{32}q_3 + y_2 &= q_2 \\ a_{13}q_1 + a_{23}q_2 + a_{33}q_3 + y_3 &= q_3 \end{aligned} \tag{L4}$$

Eq.(L4) can be written in the following matrix form:

$$\mathbf{Aq} + \mathbf{y} = \mathbf{q} \tag{L5}$$

where $\mathbf{A} = [a_{ij}]$ $(i, j = 1, 2, 3)$, $\mathbf{q} = [q_1, q_2, q_3]^T$ and $\mathbf{y} = [y_1, y_2, y_3]^T$.

The solution of Eq.(L5) is obtained in the following Leontief inverse matrix solution:

$$\mathbf{q} = [\mathbf{E} - \mathbf{A}]^{-1}\,\mathbf{y} \tag{L6}$$

However, since we cannot practically determine a representative single produced good in an industry, the input output table is made with monetary value expression. Transforming each element in Fig. 2 into the price $\times$ quantity expression considering Assumptions a1 to a6 gives Fig.9 .

industry	1	2	3	a	b
1	p_1q_{11}	p_1q_{12}	p_1q_{13}	p_1y_1	p_1q_1
2	p_2q_{21}	p_2q_{22}	p_2q_{23}	p_2y_2	p_2q_2
3	p_3q_{31}	p_3q_{32}	p_3q_{33}	p_3y_3	p_3q_3
a	wq_{V1}	wq_{V2}	wq_{V3}		
b	p_1q_1	p_2q_2	p_3q_3		

Fig.9 Price × quantity expression

where wq_{Vj} is the gross value added: e.g. when we assume wq_{Vj} as the total wages in industry i, q_{Vj} means the number of laborers; w means wages per unit labor i.e. labor price.

The relationship between rows 1, 2 and 3 in Fig.9 is the same as in Eq.(L1), therefore the result of an output analysis in production gives the same form equations with either a monetary or quantitative unit. However, we must not regard that Eq.(L6) or Eq.(7) is the solution of the open model output equation, because the solution includes the multiplier effect due to the Leontief inverse matrix. The correct solution of the input output analysis is the single Eq.(17), in which the multiplier effect is not included.

3.2 Investigation of the Leontief price model analysis

The price model equation is obtained by an operation toward the vertical (column) direction in Fig.9. Taking each quantity q_i in row b in Fig.9 as 1 gives Fig.10. For example, q_{11}/q_1 means an extended quantity at the matrix element $(1, 1)$.

industry	1	2	3
1	$p_1(q_{11}/q_1)$	$p_1(q_{12}/q_2)$	$p_1(q_{13}/q_3)$
2	$p_2(q_{21}/q_1)$	$p_2(q_{22}/q_2)$	$p_2(q_{23}/q_3)$
3	$p_3(q_{31}/q_1)$	$p_3(q_{32}/q_2)$	$p_3(q_{33}/q_3)$
a	$w(q_{V1}/q_1)$	$w(q_{V2}/q_2)$	$w(q_{V3}/q_3)$
b	$p_1 \cdot 1$	$p_2 \cdot 1$	$p_3 \cdot 1$

Fig.10 Price × quantity expression

The relationship between quantities and prices in Fig.10 produces the following:

$$
\begin{aligned}
(q_{11}/q_1)p_1 + (q_{21}/q_1)p_2 + (q_{31}/q_1)p_3 + w\alpha_1 &= p_1 \\
(q_{12}/q_2)p_1 + (q_{22}/q_2)p_2 + (q_{32}/q_2)p_3 + w\alpha_2 &= p_2 \\
(q_{13}/q_3)p_1 + (q_{23}/q_3)p_2 + (q_{33}/q_3)p_3 + w\alpha_3 &= p_3
\end{aligned}
\tag{L7}
$$

where $\alpha_i = q_{Vi}/q_i$.

In Eq.(L7) if we define coefficients b_{ij} (constants) as shown in Eq.(L8), we have Eq.(L9-1):

$$b_{ij} = q_{ji}/q_i \tag{L8}$$

$$b_{11}p_1 + b_{12}p_2 + b_{13}p_3 + w\alpha_1 = p_1 \quad \text{(L9-1)}$$
$$b_{21}p_1 + b_{22}p_2 + b_{23}p_3 + w\alpha_2 = p_2$$
$$b_{31}p_1 + b_{32}p_2 + b_{33}p_3 + w\alpha_3 = p_3$$

Using the symbol $\mathbf{B}$ defined in Eq.(12) gives:

$$\mathbf{BP} + w\alpha = \mathbf{p} \quad \text{(L9-2)}$$

where $\mathbf{p} = [p_1, p_2, p_3]^T$, $\alpha = [\alpha_1, \alpha_2, \alpha_3]^T$. Eq.(L9-2) gives:

$$\mathbf{p} = w\,[\mathbf{E} - \mathbf{B}]^{-1}\,\alpha \quad \text{(L10)}$$

In Eq.(L8) $\mathbf{B} \neq \mathbf{A}^T$ is clearly shown. In the matrix transformation for Eq.(L3), Leontief set up the following in place of Eq.(L9-2) :

$$\mathbf{A}^T\mathbf{p} + w\alpha = \mathbf{p} \quad \text{(L11)}$$

This is his error.

We cannot interpret that Eq.(L10) is the correct solution of the price model analysis because it is obtained from Eq.(L9-2). The solution of the input output analysis equations shown in Fig.2 is the single Eq. (17). If we regard Eq.(L10) alone as the correct solution, we will have an incorrect solution with a multiplier effect.

The fundamental cause of the error in Leontief's input output analysis results from the formulation process itself. The input output table expresses that, for example in industry 1, debit= $P_{11} + P_{21} + P_{31} + V_1$, credit= $P_{11} + P_{12} + P_{13} + Y_1$ and total debit=total credit=X. Although the numerical value of each symbol in the table is asymmetric, the array of the symbols is symmetrical. In the process of deriving equations, the symmetrical property of symbols in horizontal and vertical directions must be preserved in the same manner as in the operations in Eq.(2) and Eq.(9). As a result, the same shape equations as Eq.(5) and Eq.(12) are derived; the notations of $\mathbf{A}$ in Eq.(3) and $\mathbf{B}$ in Eq.(10) must be used due to the asymmetric property of the numerical values in $\mathbf{A}$ and $\mathbf{B}$. In the Leontief input output analysis, the symmetric property in the formulation process in the horizontal and vertical directions is not preserved. Consequently, even if the correct Eq.(L9-2) is used in place of Eq.(L11), the error of the input output analysis method is not eliminated.

As analyzed above, the relationships between prices and quantities for produced goods are given as trivial solutions: one side, the solutions are $\Sigma_i V_i$(= price×quantity) = $\Sigma_i Y_i$ (= price×quantity); the other side, the solutions of price and quantity for P_{ij} are indeterminate because we have an infinite number of P_{ij} (= price×quantity) which satisfies Eq.(17). This is confirmed in Numerical calculation example 3.

3.3 Economic values of P matrix elements in economic analysis systems

Here the Leontief input output analysis is interpreted from the viewpoint of the CF matrix analysis and it is explained where economic values lie in the **P** matrix elements.

In the Leontief input output analysis models there are two systems; the **closed system** and the **open system**. As shown in Fig.1, the open system is that in which the transactions between industrial intermediate goods are inside the dotted line frame, and both GDP(+IM) and GVA(+IM) are outside the frame. The closed system is that in which the intermediate goods, GDP(+IM) and GVA(+IM) are inside the frame.

In the closed system households and governments etc. are regarded as industries, hence all industries are placed inside the dotted line frame. For example, in the households sector as the final consumers, labor is output and consumption goods are output. At first glance this may seem unnatural. However, if we know that, in a households account, debit = Purchasing consumption goods + Saving + Transfer and credit = Wages + Transfer, we can interpret from the CF matrix analysis that the closed system is merely the one where the households account made from both one row (credit) and one column (debit) is added to the industrial production accounts.

In the same way, we can separate off the financial account from the original industries account (a CF matrix table) and can place the financial account next to the remaining original account again. In the closed system all variables are endogenous; the system is an accounting system which becomes the basis of economic analyses. We can logically express all accounts comprising every enterprise and household in a nation in the form of one table (the CF matrix table) as the closed system.

There are many variables relating to the closed system. From Leontief's analysis, we select and fix some variables from the closed system as fundamental value factors for economic activities. If we regard the variables as the exogenous variables, we can set up simultaneous equations comprising the exogenous variables and the remaining endogenous variables. We select GDP(+IM) and GVA(+IM) as exogenous variables for each sector. This is the open system.

At this time, the endogenous variables logically do not hold the economic values held by exogenous variables; therefore, the former variables must merely express economic transactions between economic activity clients with or without cash flows. The cash in this condition means a tool for exchanging real assets; or means a human promise as words such as entrusted money or loaned money.

Based on this logic, let us explain where economic values lie in the **P** matrix elements. The image of the GVA(+IM) formulated by Leontief is as follows for Fig.1 :

- The open and closed systems are shown in Fig.11. The mark "×" denotes final goods. From Numerical calculation example 3, if we fix **Y** and **V**, all the **P** elements are indeterminate. If we give **Y** and **V** whole economic values, the **P** elements have no economic value, and so **P** merely shows a form in economic transaction structures or industrial layered structures. Giving the **P** elements further economic values is impossible at least in production analyses.

- As another analytical model we can give both P_{ij} and Y_{ij} production values. We call the model the **real transaction system**, an image of which is shown in Fig.12. Through any enterprise production stage, a part of the GVA is included in any production goods added by each enterprise. In Figs.11 and 12 there is really no case where the whole of one of the **P** elements becomes a final product.

In short, the input output table analysis provides an analytical model to research a production process. If we concentrate the whole production value on the final products **Y** and **V**, no production value is included in **P**. If we concentrate the whole production value in **P**, no production value is included in **Y** and **V**. In fact, a partial gross value added is given through each production process. The aggregated production values are **Y** or **V**.

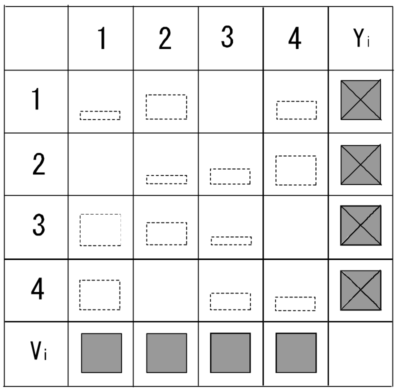

Fig.11 Open and Closed models

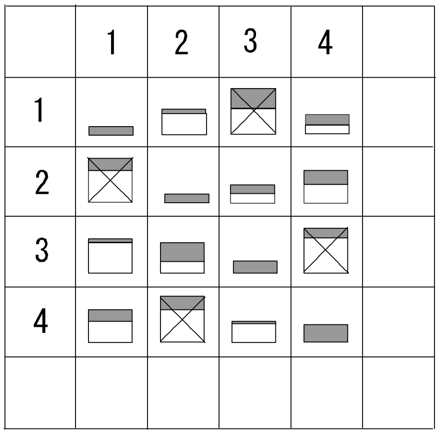

Fig.12 Real transaction system

3.4 True meaning of the multiplier effect in the Leontief inverse matrix operation

3.4.1 Affine transformation

Understanding that economic values are not included in **P**, the mathematical meaning of **P** will be considered. First a simple definition will be introduced. Refer to Fig.2 where **P** , **Y** , **V** and **X** are included. Between those variables there are Eq.(5) and Eq.(12) as basic equations, and Eq.(17) $\mathbf{Y} = \mathbf{HV}$ (or Eq.(18)) as a linear combination equation.

Let $\mathbf{V} = [V_1, V_2, V_3]^T$, $\mathbf{Y} = [Y_1, Y_2, Y_3]^T$, and $\mathbf{H} = [H_{ij}]$ $(i, j = 1, 2, 3)$. Given an invertible matrix **H**, $\mathbf{Y} = \mathbf{HV}$ is a **regular Affine transformation** which maps **V** to **Y**. In addition, since there is no translation in Eq.(17), this Affine transformation consists of rotation, scaling, and shearing for vectors. An Affine transformation is usually applied in 3 dimensional graphics, film making, physics and engineering etc.

In short, $\mathbf{Y} = \mathbf{HV}$ merely shows a mapping (functional) relationship between exogenous variables **V** and **Y** given as constants. There exist, in this relationship, the mappings $\mathbf{X} = [\mathbf{E} - \mathbf{A}]^{-1}\mathbf{Y}$ and $\mathbf{X} = [\mathbf{E} - \mathbf{B}]^{-1}\mathbf{V}$. These mappings merely join **V** to **Y** making **X** lie between them. Here the only solid relationship is that economic value of **V** equals that of **Y**. **X** can vary depending on national industrial structures such as a layered industrial structure, including small and medium companies; or a structure comprising of giant state-owned enterprises. Consequently, **H**'s structure infinitely exists as long as Account 1 holds. We cannot mathematically add economic values to **P** depending on an uncountable number of **H**. It is shown in Numerical calculation example 3 that the combinations of **P** elements which map **V** to **Y** are infinite.

3.4.2 Graphic expression of Affine transformation

An example of an Affine transformation for a 2 dimensional model is demonstrated in the following:

	1	2	Y_i	X_i
1	20	40	40	100
2	70	30	30	130
V_i	10	60		
X_i	100	130		

Fig.13 2 dimensional CF matrix model

$$\mathbf{Y}=\begin{bmatrix}40\\30\end{bmatrix}\quad \mathbf{V}=\begin{bmatrix}10\\60\end{bmatrix}\quad \mathbf{X}=\begin{bmatrix}100\\130\end{bmatrix}\quad \mathbf{P}=\begin{bmatrix}20&40\\70&30\end{bmatrix}$$

$$\mathbf{A}=\begin{bmatrix}20/100&40/130\\70/100&30/130\end{bmatrix}\quad \mathbf{AX}=\begin{bmatrix}60\\100\end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix}20/100&70/130\\40/100&30/130\end{bmatrix}\quad \mathbf{BX}=\begin{bmatrix}90\\70\end{bmatrix}$$

Fig.14 shows the Affine transformation for Fig.13.

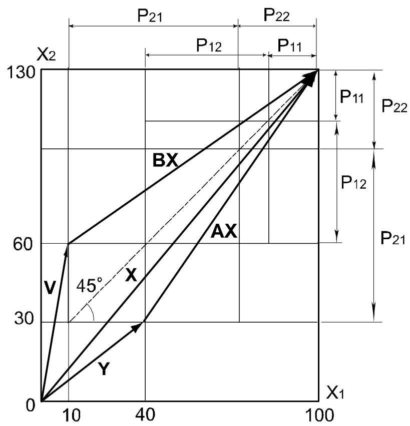

Fig. 14 Affine transformation

The relationship with the table elements in Fig. 14 will be shown in another type chart. Fig.14 has been obtained from Eq.(5)and Eq.(12). We can transform Account 1 into Account 2 from Eq.(2), Eq.(3), Eq.(9) and Eq.(10).

Account 2

Sector 1		Sector 2		Sector 3	
Dr.	Cr.	Dr.	Cr.	Dr.	Cr.
$b_{11}X_1$	$a_{11}X_1$	$b_{21}X_1$	$a_{21}X_1$	$b_{31}X_1$	$a_{31}X_1$
$b_{12}X_2$	$a_{12}X_2$	$b_{22}X_2$	$a_{22}X_2$	$b_{32}X_2$	$a_{32}X_2$
$b_{13}X_3$	$a_{13}X_3$	$b_{23}X_3$	$a_{23}X_3$	$b_{33}X_3$	$a_{33}X_3$
V_1	Y_1	V_2	Y_2	V_3	Y_3
X_1	X_1	X_2	X_2	X_3	X_3

From Account 2 we have the following:

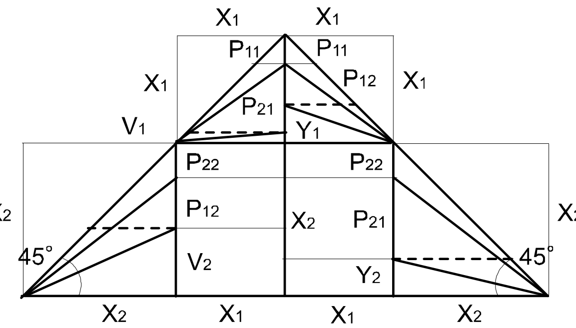

Fig.15 Chart of sales versus gross value added, with **P**

3.5 Achievements of the Leontief input output analysis

The author has shown that the Leontief input output analysis has a large imperfection from the present viewpoint. However, the author himself appreciates the fact that Leontief presented the original input output analysis method and applied it to practical economic analyses. In the author's opinion, Leontief is an economist equal to L.Walras, J.M.Keynes and J.Shumpeter, who through their works in the production analysis area, left the greatest economic achievements in the history of economics. The reasons are as follows:

- Leontief's greatest achievement was finding a methodology which enables economic analyses using a matrix manipulation different from the 3×3-SNA account form in the SNA accounts. Using the input output table halves data included in a set of accounts, enables consistent processing for the data and, as a result, simplifies economic analyses.
- The input output analysis presented an analytical method in which a set of economic data were divided into endogenous and exogenous data. It revealed the meaning of intermediate goods matrix **P**.
- The input output table can join all economic data from one enterprise production account in a micro-economy to a set of national production accounts in a macro economy; thereby all data can be theoretically consistent with each other without any data exclusion through the process .
- For any given data we can analyze the microeconomic behavior of a targeted industry preserving other industries' macroeconomic behavior properly scaling the number of industrial sectors and enterprises included in a sector depending on an analytical need.

- Since the input output table structure is a 2 dimensional expression for many accounts, the input output analysis is suitable for use with a 45degree chart.
- Although Leontief failed, it has become clear that in an economic analysis we should distinguish the existential condition of gross value added in goods. This will have an effect on the reexamination of the Walrasian general equilibrium theory.
- Using the CF matrix analysis we can analyze economies making both financial credit and real goods consistent with each other. From this point, there is a possibility of being presented with a new economic theory to connect credit to real goods.

4 Conclusions to Part 1

The author proposed the CF matrix analysis method in place of the conventional Leontief Input-output analysis and obtained the following conclusions:

1. There are the basic production equations (named the sales transaction equation by the author) shown in Eq.(5) or Eq.(7) in Leontief Input-output analysis. The author (Hayashi) presents a new basic equation; the purchase transaction equation named by him, shown in Eq.(12) or Eq.(14), which is on a par with Eq.(7). He further presents the joined transaction equation which joins Eq.(7) and Eq.(14).
2. According to Numerical calculation example in 2.4.3, when we take vectors $\mathbf{Y}$ and $\mathbf{V}$ in Eq.(17) as exogenous variables, we can determine any element in the intermediate goods matrix $\mathbf{P}$ to have any value, so the elements in $\mathbf{P}$ are indefinite. Therefore, $\mathbf{P}$ does not include any factor value in the gross value added. $\mathbf{P}$ merely expresses a transaction form from $\mathbf{Y}$ to $\mathbf{V}$. Eq.(17) shows mathematically an Affine transformation which maps $\mathbf{V}$ to $\mathbf{Y}$.
3. Consequently, if we use only Eq. (7) for the input output analysis, said analysis should be considered an unfinished theory, though we will not say it is incorrect.
4. Leontief's price model analysis is incorrect because it is inconsistent with Eq.(14).
5. As a result, in the multiplier effect equation in the input output analysis or in the Leontief inverse matrix solution, no multiplier effect to either the final demands or national income is found.

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